III. **Solution by M. C. STEVENS, M. A., Mathematical Department, Purdue University, Lafayette, Indiana; HENRY HEATON, M. Sc., Atlantic, Iowa; JOHN B. FAUGHT, A. M., Instructor in Mathematics in Indiana University, Bloomington, Indiana; and J. C. GREGG, A. M., Brazil, Indiana.**

Put $y=vx$ and we have

$$v+x\frac{dv}{dx}=\frac{vx^2-v^2x^2}{x^2+vx^2}=\frac{v-v^2}{1+v}=v-\frac{2v^2}{1+v}.\quad \text{Whence, } \frac{1+v}{v^2}dv+\frac{2dx}{x}=0.$$

Integrating, $-1/v+\log(vx^2)+C=0$, or $x/y=\log(xy)+C$, and $x=y\log(xy)+Cy$.

The $C$ should not be omitted unless the conditions of the question giving rise to the equation are such as to make it zero.

IV. **Solution by H. C. WHITTAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

Let $y=x^pv^q$ and substitute in the given equation and we obtain

$$\frac{dv}{dx}=\frac{(1-p)v-(1+p)x^{p-1}v^{q+1}}{q^x(1+x^{p-1}v^q}.$$

This will reduce to a simple form if we take $p=1$ and $q=-1$, giving

$$\frac{dv}{dx}=\frac{2}{x(1+v^{-1})},\ \text{or } dv(1+v^{-1})=2x^{-1}dx.$$

$v+\log v=\log x^2$ ; $x/y+\log(x/y)=\log x^2$.
$x/y=\log x^2-\log(x/y)=\log(xy)$, whence $x=y\log(xy)$.

54. **Proposed by J. SCHEFFER, A. M., Hagerstown, Maryland.**

A certain solid has a square, side$=a$, for its base, and all parallel sections are squares, the two sections through the middle points of the opposite side of the square are semi-circles, however. Find surface, volume, and center of gravity of each.

I. **Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.**

The length of a side of a parallel section distant $x$ from the base is $(a^2-4x^2)^{\frac{1}{2}}$. If $dx$ be the distance between two parallel sections, the distance between two corresponding sides is $adx/(a^2-4x^2)^{\frac{1}{2}}$. Hence the surface

$$S=4\int_0^{\frac{1}{2}a}adx=2a^2\ ;\ \text{the volume } V=\int_0^{\frac{1}{2}a}(a^2-4x^2)dx=\tfrac{1}{3}a^3\ ;$$

the distance of the center of gravity of the surface from the base is

$$\frac{1}{2a^2}\int_0^{\frac{1}{2}a}axdx=\tfrac{1}{8}a\ ;$$

and the distance of the center of gravity of the volume from the base is

$$\frac{3}{a^3}\int_0^{\frac{1}{2}a} x(a^2-4x^2)dx=\tfrac{3}{16}a.$$

II. Solution by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering in the State A. M. College, College Station, Texas.

Take the intersection of the planes of the circular sections as the axis of $z$, the origin being in the center of the base. Then since the radius of each circle is $\frac{1}{2}a$ we shall have for the projection of one fourth of the elementary area intercepted between two planes parallel to the base, and distant $dz$ from each other, upon the plane of one of the circles,

$$dS.\cos\theta=\sqrt{\tfrac{1}{4}a^2-z^2}.dz,$$

where $\theta$ is the angle made by this elementary area with the plane of projection.

But $\cos\theta=\dfrac{\sqrt{\frac{1}{4}a^2-z^2}}{\frac{1}{2}a}$, and the whole surface is $S=4\int_0^{\frac{1}{2}a} adz=2a^2\ldots.(1)$.

The center of gravity of $S$ is distant from the base

$$z_1=\frac{4\int_0^{\frac{1}{2}a} azdz}{2a^2}=\tfrac{1}{4}a\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2).$$

For the volume, taking planes parallel to the base,

$$V=\int_0^{\frac{1}{2}a} 2\sqrt{\tfrac{1}{4}a^2-z^2}.2\sqrt{\tfrac{1}{4}a^2-z^2}.dz=\int_0^{\frac{1}{2}a}(a^2-4z^2)dz=a^3/3\ldots\ldots\ldots(3),$$

and its center of gravity above the base is

$$z_2=\frac{\int_0^{\frac{1}{2}a}(a^2-4z^2)zdz}{\frac{1}{3}a^3}=\tfrac{3}{16}a\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

The figure will be a cloistered arch formed by the intersection of two right semi-cylinders.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let $x^2+z^2=\frac{1}{4}a^2\ldots\ldots(1)$, $y^2+z^2=\frac{1}{4}a^2\ldots\ldots(2)$ be the equations to the cylinders which form the groin. From (1) $dz/dx=-x/z$, $dz/dy=0$.

$$S=\int\int\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}\,dxdy=8\int_0^{\frac{1}{2}a}\int_0^x\sqrt{1+\frac{x^2}{z^2}}\,dxdy$$

$$=4a\int_0^{\frac{1}{2}a}\int_0^x\frac{dxdy}{z}=4a\int_0^{\frac{1}{2}a}\int_0^x\frac{dxdy}{\sqrt{\frac{1}{4}a^2-x^2}}=4a\int_0^{\frac{1}{2}a}\frac{xdx}{\sqrt{\frac{1}{4}a^2-x^2}}=2a^2.$$

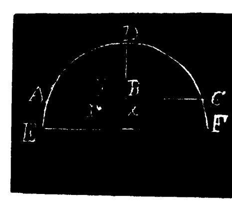
$$V=\int\int\int dzdxdy=4\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}\sqrt{a^2-4z^2}}\int_0^{\frac{1}{2}\sqrt{a^2-4z^2}}dzdxdy=\int_0^{\frac{1}{2}a}(a^2-4z^2)dz=\tfrac{1}{3}a^3.$$

$$\text{Center of graving of surface}=\frac{\int\int zdS}{\int\int dS}=\tfrac{1}{2}a\int_0^{\frac{1}{2}a}\int_0^x dxdy=\tfrac{1}{4}a.$$

$$\text{Center of gravity of volume}=\frac{\int\int\int zdzdxdy}{\int\int\int dzdxdy}=\frac{3}{a^3}\int_0^{\frac{1}{2}a}z(a^2-4z^2)dz=\tfrac{3}{16}a.$$

IV. **Solution by J. C. GREGG, A. M., Brazil, Indiana.**

Let the given figure represent a section of the solid through the middle point of two opposite sides of the base. We have $r=a/2$, and the equation of the circle $EDF$ is $x^2+y^2=r^2\ldots.(1)$, and $AC^2=(2y)^2=4(r^2-x^2)=A_x=$a section parallel to the base, and for the volume




$$V=4\int_0^r(r^2-x^2)dx=\tfrac{8}{3}r^3=\tfrac{1}{3}a^3.$$

The surface may be considered to be generated by the sides of a section parallel to the base, and we have for the surface,

$$S=4\int 2yds=4\int_0^r 2\sqrt{r^2-x^2}.\frac{rdx}{\sqrt{r^2-x^2}},\ =8r\int_0^r dx=8r^2=2a^2.$$

For the center of gravity of the volume,

$$\bar{x}=\frac{\int x(A_x)dx}{V},\ =\frac{4\int_0^r x(r^2-x^2)dx}{\frac{8}{3}r^3},\ =\frac{3}{2r^3}\int_0^r x(r^2-x^2)dx=\tfrac{3}{8}r=\tfrac{3}{16}a.$$

For the center of gravity of the *curved* surface we have,

$$\bar{x}=\frac{4\int 2xyds}{S},\ =\frac{8r\int_0^r xdx}{8r^2}=\frac{1}{r}\int_0^r xdx,\ =\tfrac{1}{2}r,\ =\tfrac{1}{4}a.$$

For the center of gravity of the *whole* surface, since the curved surface is *twice* that of the base we have, $\bar{x}=\frac{2}{3}.\frac{1}{4}a=\frac{1}{6}a$.

Also solved by *H. C. WHITAKER*, *C. W. M. BLACK*, and the *PROPOSER*.

Professors Black and Scheffer used "side$=2a$" as in Problem 47, instead of *side*$=a$, and hence their results did not agree with those in the published solutions. The results obtained were: Volume$=8a^3/3$, surface$=8a^2$, center of gravity of volume$=3a/8$, and center of gravity of surface$=\frac{1}{2}a$. See problem 42 for two additional SOLUTIONS for surface and volume. EDITOR.

# MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

38. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

A prolate spheroid of revolution is fixed at its focus; a blow is given it at the extremity of the axis minor in a line tangent to the direction perpendicular to the axis major. Find the axis about which the body begins to rotate. [From *Loudon's Rigid Dynamics*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

The general equations of motion are :

$$\left.\begin{array}{l} A\omega_x-(\Sigma mxy)\omega_y-(\Sigma mxz)\omega_z=L \\ B\omega_y-(\Sigma myz)\omega_z-(\Sigma myx)\omega_x=M \\ C\omega_z-(\Sigma mzx)\omega_x-(\Sigma mzy)\omega_y=N \end{array}\right\} \ldots\ldots\ldots\ldots\ldots (1).$$

The equation to the ellipsoid with focus as origin is $a^2y^2+a^2z^2+b^2x^2=2aeb^2x+b^4$. Now $\Sigma mxy=\Sigma mxz=\Sigma myz=0$. $\therefore$ (1) reduce to

$$\left.\begin{array}{l} A\omega_x=L \\ B\omega_y=M \\ C\omega_z=N \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$